

Table 1. Comparison of eigenvalues

	$Bi_w = 0$	0.25	0.5	1.0	2.5	5.0	10.0	$\infty$
$\lambda_0$	0.004854	0.9463	1.272	1.641	2.102	2.357	2.517	2.704
	1.333	1.536	1.681	1.878	2.173	2.363	2.496	2.667
$\lambda_1$	5.068	5.187	5.295	5.478	5.841	6.135	6.365	6.679
	5.333	5.426	5.509	5.650	5.932	6.172	6.371	6.667
$\lambda_2$	9.158	9.234	9.306	9.436	9.729	10.01	10.27	10.67
	9.333	9.399	9.459	9.568	9.814	10.06	10.29	10.67
$\lambda_3$	13.20	13.26	13.31	13.42	13.66	13.93	14.20	14.67
	13.33	13.39	13.43	13.52	13.74	13.98	14.22	14.67

The data shown in the upper row are obtained from the present power series solutions and the data in the lower row are obtained by computing the roots of equation (2).

### CONCLUSIONS

1. A larger error in the length of ice-free zone results from employing the eigenvalues computed from the asymptotic solution for the boundary condition of second or third kind. Conventional power series method should be used in calculating the first few eigenvalues.

2. The length of ice-free zone predicted by the present power series solution agrees excellently with the numerical result. The previous result [2] checks well with the present solutions when the Biot number is large. But it carries large error when the Biot number is small.

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## CONDENSATION OF BINARY MIXTURES OF MISCIBLE VAPORS

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### NOMENCLATURE

$D$ , binary diffusion coefficient;  
 $g$ , acceleration of gravity;  
 $K_L$ , thermal conductivity of the condensate;  
 $K$ , constant suction parameter expressing the strength of interfacial suction;

$M_1, M_2$ , molecular weights of the binary species;  
 $P$ , total pressure;  
 $P_1, P_2$ , vapor pressure of the pure components [5];  
 $q$ , actual local surface heat flux evaluated at  $(T_i - T_w)$ ;  
 $q_0$ , reference heat flux evaluated at  $T_i = T_\infty$ ;

- Sc, Schmidt number  $\nu/D$ ;
- T, temperature;
- x, axial coordinate placed along the condensate surface.

Greek symbols

- $\lambda$ , latent heat;
- $\mu$ , viscosity;
- $\nu$ , kinematic viscosity;
- $\rho$ , density;
- $\gamma$ , activity coefficient which takes into account deviation of the binary condensate solution from ideal behaviour [5].

Subscripts

- i, vapor-liquid interface;
- L, condensate;
- s, suction;
- W, plate surface;
- 1, the volatile component (corresponds to methanol vapor in Figs. 1 and 2);
- 2, the less volatile component (corresponds to water vapor in Figs. 1 and 2);
- $\infty$ , bulk of the vapor.

Unsubscripted properties are for vapor mixture.

In 1969 Sparrow and Marschall [1] presented a predictive theory for the condensation process of binary mixtures on a cooled vertical plate. This analysis is based upon a rigorous solution of the vapor boundary layer equations and those of the condensate film. In the present note the approximate integral method is used to solve this problem. It provides a simpler procedure to calculate the condensation rates with a reasonable accuracy. Also the effect of interfacial suction at the vapor-liquid interface is added, and the improvement in the condensation efficiency is calculated for suction velocities which obey  $v_s = K x^{-\frac{1}{2}}$ . The conservation equations of momentum and mass solved here as well as their integral representation and the appropriate profiles are similar to those presented in [2, 3]. Therefore we give here only the final results. The following equation relates the mass fraction of the volatile component in the condensate,  $\bar{W}_{1i}$ , with its equilibrium value,  $W_{1i}$ , at the vapor interface:

$$10 SGSc (1 - f_s)^2 \left[ \frac{W_{1i} - \bar{W}_{1i}}{W_{1i} - W_{1\infty}} - \frac{1}{1 - f_s} \right]^2 \times \left[ \frac{20}{21} + \frac{Sc}{1 - \frac{W_{1i} - W_{1\infty}}{W_{1i} - \bar{W}_{1i}} \frac{1}{1 - f_s}} \right] + \frac{8}{S^2 GSc (1 - f_s)^2} \times \left( \frac{W_{1i} - W_{1\infty}}{W_{1i} - \bar{W}_{1i}} \right)^2 \left[ \frac{5S}{28} - \phi \frac{(W_{1i} - W_{1\infty})}{3} \right]$$

$$- \frac{100}{21} \left( \frac{W_{1i} - W_{1\infty}}{W_{1i} - \bar{W}_{1i}} \right) \left[ \frac{W_{1i} - \bar{W}_{1i}}{W_{1i} - W_{1\infty}} - \frac{1}{1 - f_s} \right] + \frac{2}{1 - f_s} \left( \frac{W_{1i} - W_{1\infty}}{W_{1i} - \bar{W}_{1i}} \right) - 8 Sc = 0 \quad (1)$$

where

$$S = \frac{(T_i - T_w) K_L}{\lambda \mu_L}; G = \frac{\mu_L \rho_L}{\mu \rho} \text{ and } \phi = \frac{M_1 - M_2}{M_1 - W_{1\infty} (M_1 - M_2)} \quad (2)$$

$f_s$  is the ratio between the suction rate and the total vapor flux reaching the interface and is given by,

$$f_s = \frac{K}{S^{\frac{1}{2}} G^{\frac{1}{2}} (g\nu^2/4)^{\frac{1}{4}}} \quad (3)$$

For high condensation rates, namely for large differences of  $(T_\infty - T_w)$ , one may neglect the buoyancy force and treat the problem in a one dimensional fashion. Equation (1) is then replaced by:

$$\bar{W}_{1i} (1 - f_s) = W_{1\infty} - f_s W_{1i} \quad (4)$$

The physical properties are those of the binary mixture and were evaluated as detailed in [1]. The vapor-liquid equilibria relationship needed to complete the formulation may be expressed by:

$$\frac{W_{1i}}{W_{1i} + (1 - W_{2i})(M_1/M_2)} = \frac{\gamma_1(\bar{W}_{1i})P_1(T_i)}{P} \times \frac{\bar{W}_{1i}}{\bar{W}_{1i} + (1 - \bar{W}_{1i})(M_1/M_2)} \quad (5)$$

Solution for the condensation rate,  $q = \lambda \rho S^{\frac{1}{2}} G^{\frac{1}{2}} (g\nu^2/4x)^{\frac{1}{4}}$ , is obtained as follows. We guess an interface temperature  $T_i$

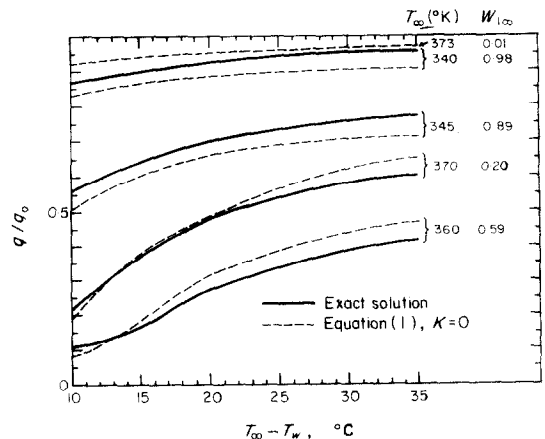


FIG. 1. Heat transfer results. Comparison of equation (1) with exact solution results ([1], Fig. 3).

and solve equations (1) and (5) by trial and error for  $W_{1i}$  and  $W_{2i}$  where  $W_{2i} = 1 - W_{1i}$ . Then  $W_{2i}$  is calculated from equation (5) provided subscript 1 is replaced by 2, excluding the molecular weight ratio. The correct  $T_i$  is obtained once  $W_{1i} + W_{2i} = 1$ .

In Fig. 1 exact solution for the heat transfer efficiency,  $q/q_0$ , given by Sparrow and Marschall [1] are compared with those obtained from equation (1) for methanol-water mixtures. It may be observed that the maximum deviation be-

tween the results is around 10 per cent. Hence, equation (1) is a reasonable solution to the problem and is suggested for practical application. In Fig. 2 it is observed that interfacial suction may increase the condensation efficiency, and in particular for low values of the thermal driving force. This is due to the increase in  $T_i$  as compared to the case where suction is absent. In general, however, the improving in condensation due to suction in this case seems to be somewhat less attractive as compared to the effect of suction in the presence of noncondensable gases [4].

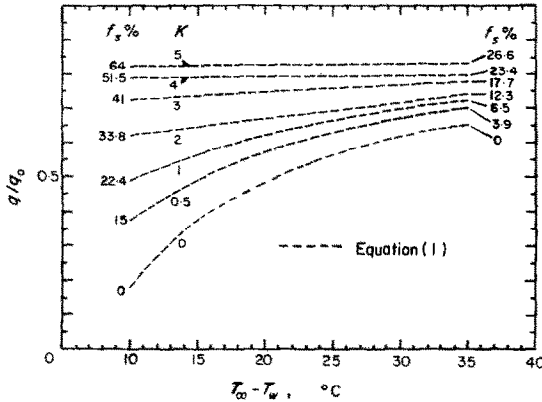


FIG. 2. Effect of interfacial suction on the heat transfer at 370°K and 760 mm Hg.

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## HEMISPHERICAL REFLECTIVITY AND TRANSMISSIVITY OF AN ABSORBING, ISOTROPICALLY SCATTERING SLAB WITH A REFLECTING BOUNDARY

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### INTRODUCTION

THE REFLECTION and transmission of radiation by a semi-transparent medium are affected by the absorption and scattering properties of the medium below the surface, the angular distribution of the incident radiation and the reflection characteristics of the bounding surfaces. A

number of investigations have been reported in the literature on the determination of radiative properties of semi-infinite and finite plane-parallel medium respectively for the case of transparent boundaries. The mathematical techniques developed by Chandrasekhar [2] have been used by several investigators [3-7] to investigate the transmission